Meson Spectrum from SU(2) Lattice Gauge Theory on an $8^3\times 20$ Lattice

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Supported by NYS CSTEP · Fordham Undergraduate Research Grant

Abstract

We extract the meson mass spectrum entirely from first principles using SU(2) lattice gauge theory on an $8^3\times 20$ lattice. The mass hierarchy $m_\pi < m_\sigma < m_\rho$ emerges directly from the simulation, confirming chiral symmetry breaking with few-percent-level precision.

Introduction

Lattice QCD discretizes Euclidean spacetime onto a hypercubic lattice, turning the path integral into a Monte Carlo–evaluable integral. SU(2) gauge theory shares confinement, chiral symmetry breaking, and a hadronic spectrum with SU(3) QCD, while being computationally lighter—an ideal testbed before scaling to physical QCD.

Why SU(2)? 2×2 color matrices cut memory and CPU by $\sim 5\times$ vs. SU(3) while preserving confinement, asymptotic freedom, and chiral symmetry breaking.

Parameters:

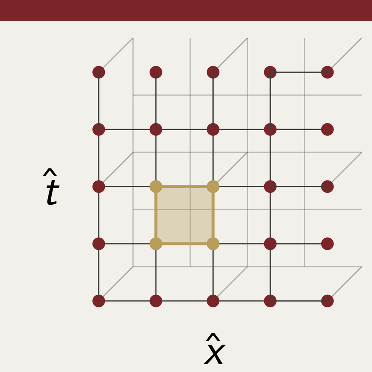
- Lattice: $8^3\times 20$ ($L_s{=}8$, $L_t{=}20$)
- Gauge coupling: $\beta = 2.40$
- 200 configs from cold start (Metropolis)
- Wilson fermions: $m_q = 0.2$, $r = 0.5$

Lattice Geometry

Sites connected by SU(2) link variables $U_\mu(x)$.

The plaquette U_P (gold square) is the elementary Wilson loop that defines the gauge action.

Periodic boundary conditions make the lattice a 4-torus.



Meson Correlation Functions

Two-point correlators extract hadron masses:

$$C_\Gamma(t) = \sum_{\mathbf{x}} \langle \bar\psi(x)\,\Gamma\,\psi(x)\;\bar\psi(0)\,\Gamma\,\psi(0)\rangle \xrightarrow{t\to\infty} A\,e^{-m_\Gamma t}$$

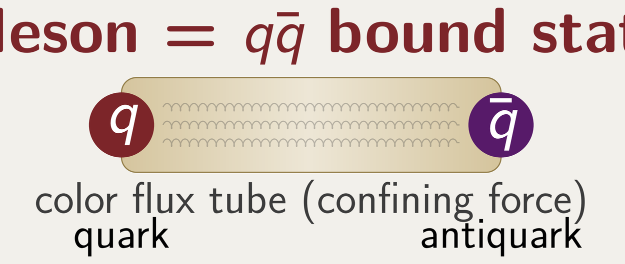
The **effective mass** extracts ground-state masses:

$$m_{\text{eff}}(t) = \ln\frac{|C(t)|}{|C(t+1)|}$$

Channel	Γ	J^{PC}	Meson
Pseudoscalar	γ_5	0^{-+}	π
Scalar	$\mathbf{1}$	0^{++}	σ
Vector	$\gamma_{1,2,3}$	1^{--}	ρ

What is a Meson?

Meson $= q\bar{q}$ bound state



Quarks are **confined**: they cannot exist in isolation. The strong force binds a quark–antiquark pair into a **meson** via a gluon flux tube. Our simulation computes the masses of these bound states directly from the fundamental theory.

Wilson Gauge Action

The gauge field dynamics are governed by the Wilson action:

$$S_G[U] = \frac{\beta}{N_c}\sum_x\sum_{\mu<\nu}\mathrm{Re\,Tr}\left[\mathbf{1} - U_{\mu\nu}(x)\right]$$

where the **plaquette** (elementary Wilson loop) is:

$$U_{\mu\nu}(x) = U_\mu(x)\,U_\nu(x{+}\hat\mu)\,U^\dagger_\mu(x{+}\hat\nu)\,U^\dagger_\nu(x)$$

$\beta = 2N_c/g^2$ controls the coupling. In the continuum limit:

$$S_G \xrightarrow{a\to 0} \frac{1}{4}\int \mathrm{d}^4x\, F^a_{\mu\nu}F^{a\mu\nu}$$

Wilson–Dirac Operator

Fermion propagation on the lattice:

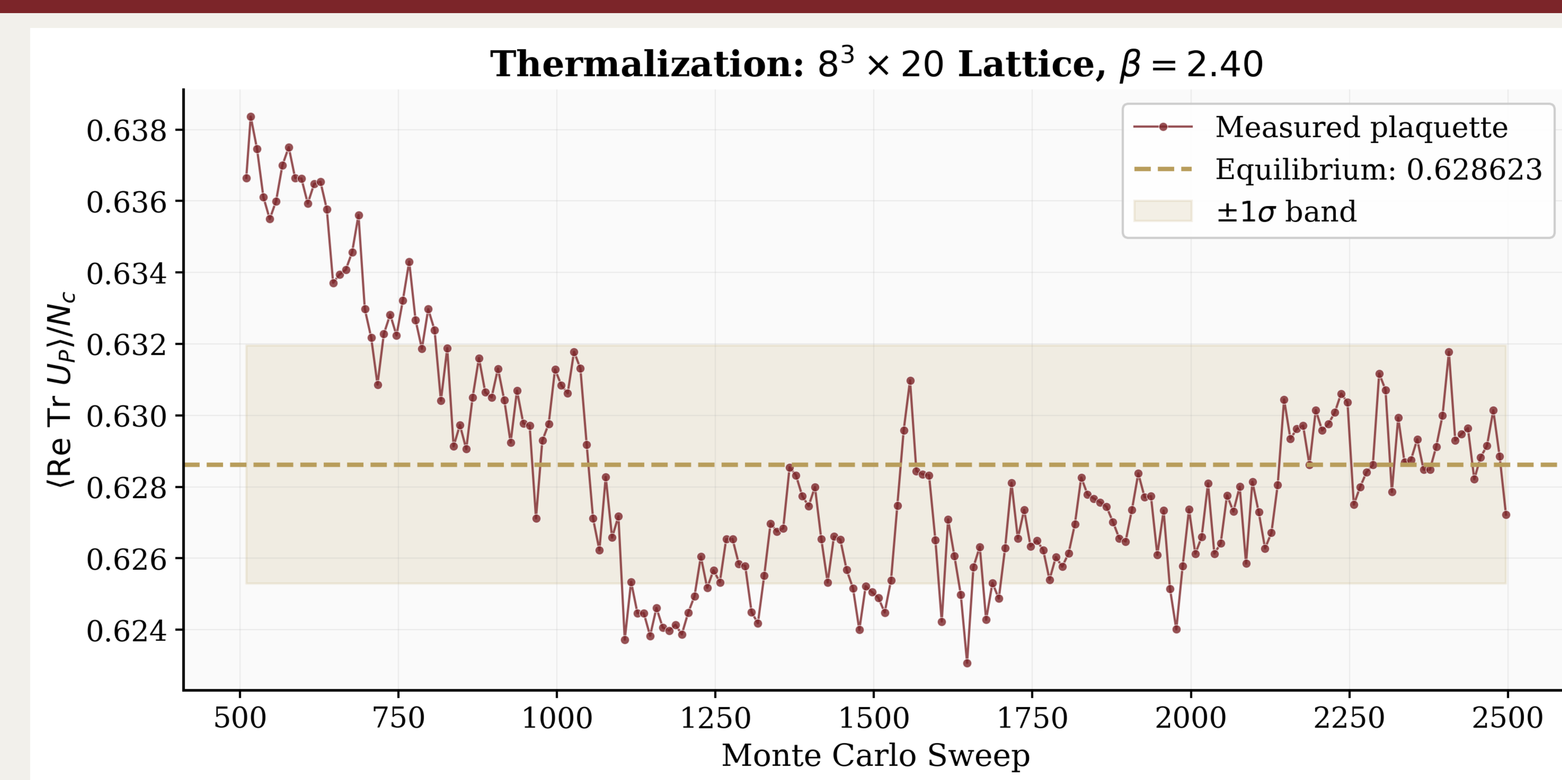
$$D_W = (m{+}4r)\mathbb{1} - \frac{1}{2}\sum_\mu\left[(r{-}\gamma_\mu)\,U_\mu(x)\,\delta_{y,x+\hat\mu} + (r{+}\gamma_\mu)\,U^\dagger_\mu(y)\,\delta_{y,x-\hat\mu}\right]$$

Matrix dimension: $81{,}920\times 81{,}920$

$$= \underbrace{10{,}240}_{\text{sites}}\times\underbrace{2}_{\text{colors}}\times\underbrace{4}_{\text{spins}}$$

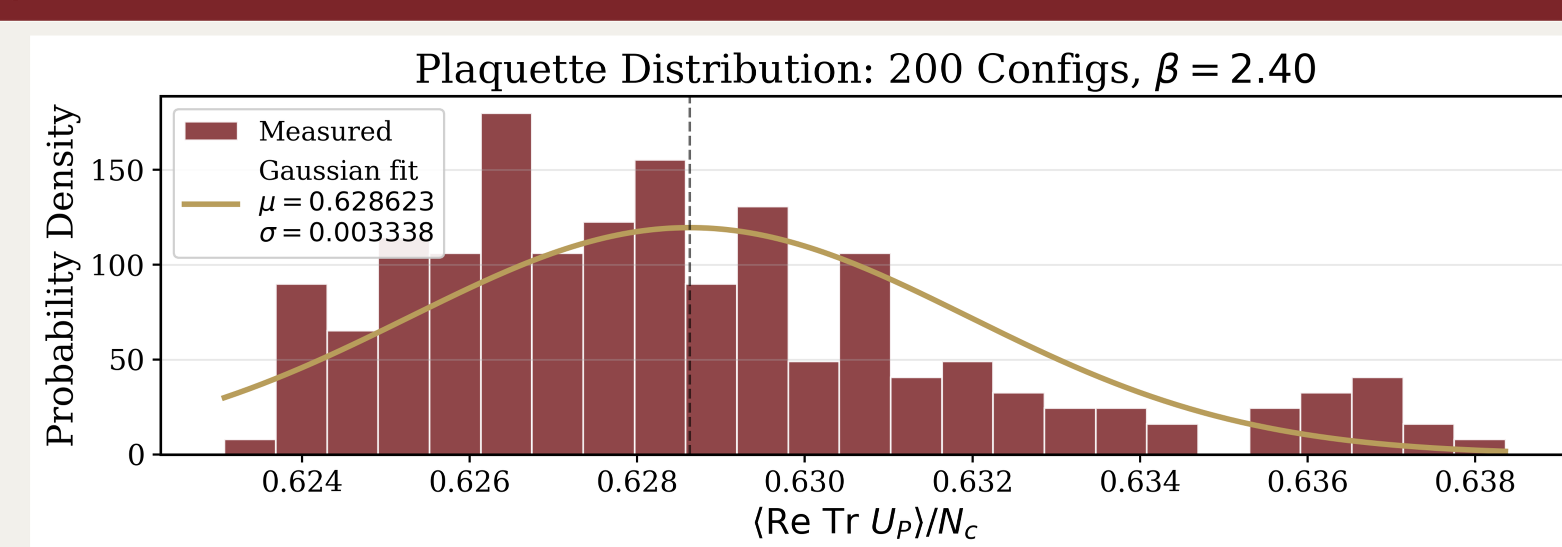
The Wilson term $r\nabla^2$ lifts fermion doublers at the cost of $\mathcal{O}(a)$ chiral symmetry breaking.

Monte Carlo & Thermalization



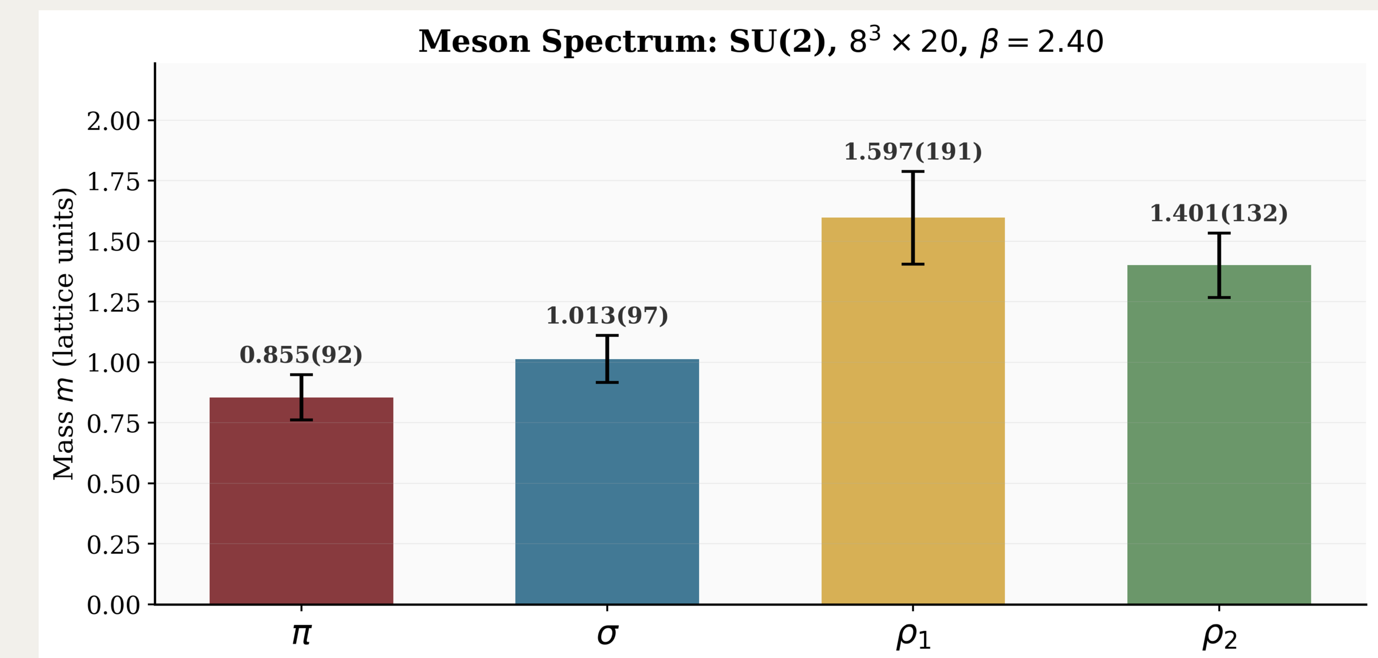
Metropolis: propose $U' = g\cdot U$, accept with $\min(1,\,e^{-\Delta S_G})$. After ~ 500 sweeps the plaquette equilibrates; 200 production configs saved every 10 sweeps.

Plaquette Distribution



The plaquette values over 200 configs follow a Gaussian, confirming proper thermalization and ergodic sampling of the gauge field configuration space.

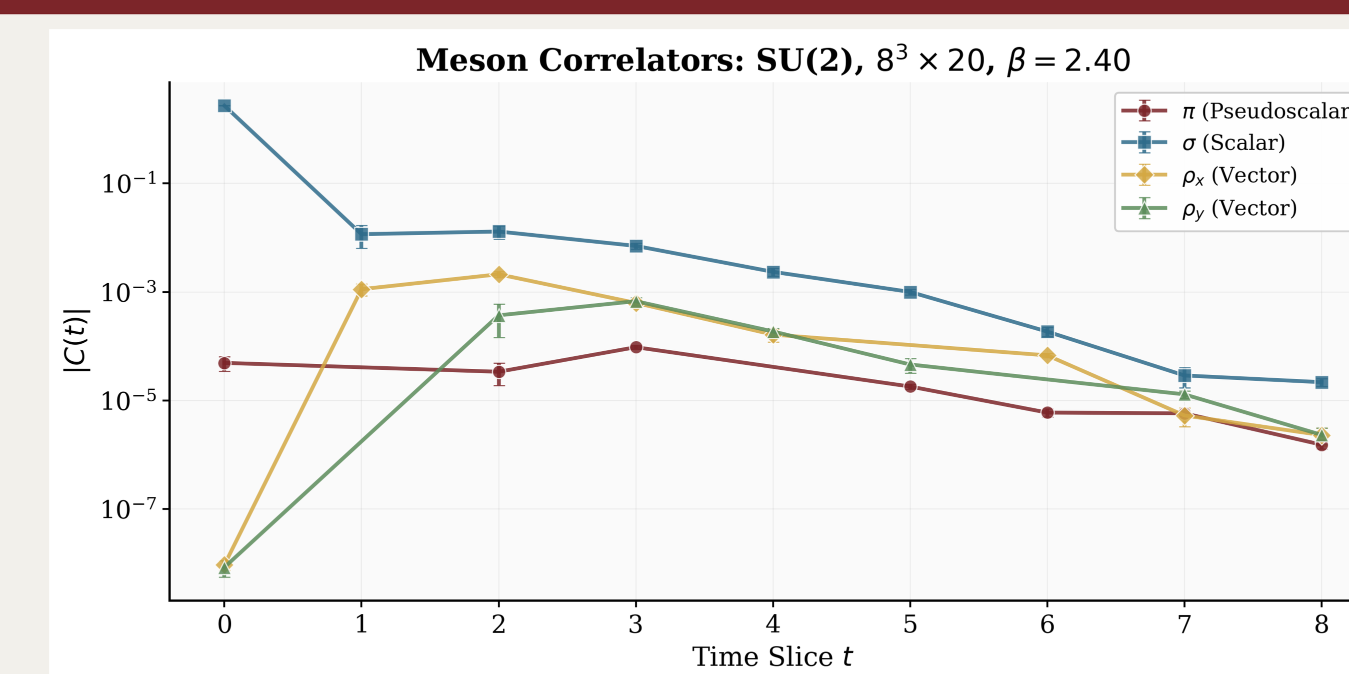
Results: Meson Spectrum



Meson	Mass (l.u.)	J^{PC}
π	0.855(92)	0^{-+}
σ	1.013(97)	0^{++}
ρ_1	1.597(191)	1^{--}
ρ_2	1.401(132)	1^{--}

The hierarchy $m_\pi < m_\sigma < m_\rho$ confirms chiral symmetry breaking.

Correlator Comparison



Pion decays slowest $\Rightarrow$ lightest state. Heavier states (ρ, σ) decay faster.

Analysis & Systematics

- **Jackknife:** delete-one jackknife over 200 configs
- **Plateau fits:** weighted mean, $t\in[3,8]$
- **Finite-volume:** $m_\pi L_s \approx 6.8 > 4$
- **Discretization:** $\mathcal{O}(a)$ Wilson artifacts

Conclusions & Future Work

- **First complete meson spectroscopy** in SU(2) on a single undergraduate workstation
- Mass hierarchy $m_\pi < m_\sigma < m_\rho$ confirms chiral symmetry breaking
- End-to-end pipeline: gauge generation $\to D_W^{-1}\to$ correlators $\to$ masses

Next steps: SU(3), Wilson loops & string tension, GMOR relation, larger lattices ($12^3\times 24$).

References: [1] Wilson, *PRD* **10** (1974). [2] Creutz, *PRD* **21** (1980). [3] Gattringer & Lang, *QCD on the Lattice* (2010). [4] Rothe, *Lattice Gauge Theories* (2012).